Development of a code for the prediction of wall temperature and void fraction in non-uniform heated water channel

Riccardo Boccelli

Politecnico di Milano

Abstract

Water technology is widely adopted in several contexts, for example in nuclear plants, where it is important to correctly predict temperature and void fraction axial profile for design and safety purposes. This work consist in the development of a script able to predict the quantities reported above for non uniform heated channel. In this first stand alone version, the code require as input the shape of the heat flux, inlet fluid properties and some other geometry characteristics, and is able to describe subcooled liquid region, subcooled boiling region, saturated boiling region, liquid deficient region and superheated vapor region.

Nomenclature:
Γ - Mass flow rate
G - Mass flux
Ω - Fluid passage area
S - Perimeter
r - Radius
D_h - Hydraulic diameter
g - Gravity acceleration
ρ - Density
ν - Kinematic viscosity
σ - Surface tension
h - Fluid enthalpy
T - Temperature
p - Pressure
α - Heat transfer coefficient
$\bar{\alpha}$ - Cross-section average void fraction
$\bar{x}$ - Cross-section average Quality
$\bar{u}$ - Cross-section average velocity
J - Mixture velocity
q''_s - Heat flux
k - Thermal conductivity
c_p - Specific heat at constant pressure

Subscripts:
b - Bulk
l - Liquid or liquid alone
lo - Liquid only
g - Vapor
go - Vapor only
tp - Two phases
s - Surface
th - Thermodynamic equilibrium
f - Friction
lg - Gas minus liquid property

I. Introduction

Flow boiling can be a desired pehomena such in Boiling Water Reactor (BWR) and in Steam Generator (SR) or an unwanted condition like in Pressurized Water Reactor (PWR) . In any case is important to predict such a phenomena and its consequences, both for design purposes (BWR and SG) and for safety analysis (PWR). The aim of this work is to develop a code able

to predict wall temperature and fluid properties along vertical pipes and rod bundles.
The system is schematized in figure 1:

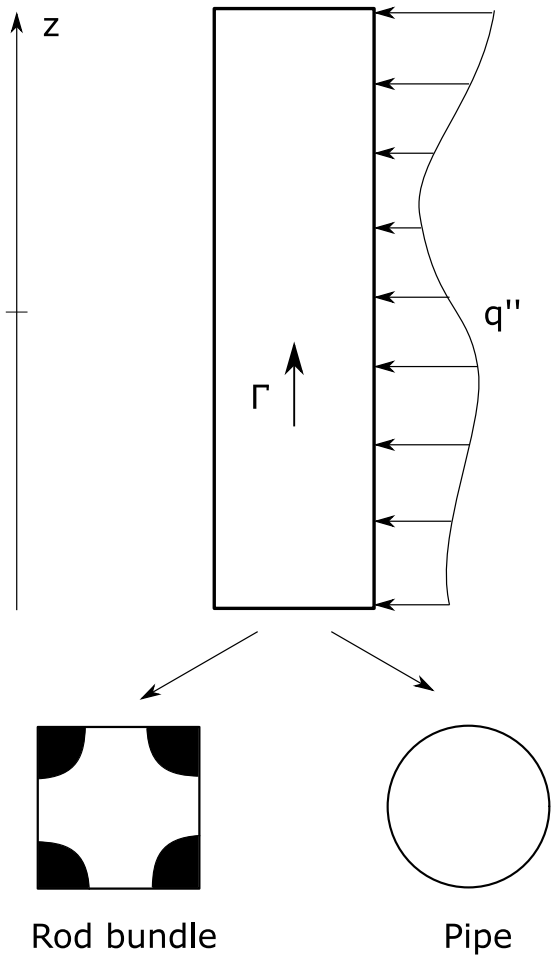

Figure 1: *Scheme of the system*

This paper is divided in few sections: first an overlook of the fluid bulk properties such as temperature, enthalpy and void fraction. Then all the correlations adopted for the heat transfer coefficient are listed. After that is explained how the code catch the critical heat fluxes and in the end are reported some general comments and a short guide which explain how to use this script.

II. Fluid temperature and void fraction

The fluid enter in the domain subcooled, if pressure drops are small compared to the system pressure, enthalpy can be evaluated by the following energy balance equation:

$$\Gamma \frac{dh_b}{dz} = q'' S \tag{1}$$

Since temperature is function of pressure and enthalpy, we can make use of XSteam Matlab function to obtain it along the whole pipe.
Since the wall temperature is higher then the bulk temperature, nucleation starts before the saturation point, the subcooled boiling region can be divided in two additional parts. The delimitation state the point where bubbles start to detach from the surface and from this point on the void fraction start becoming important. Saha and Zuber developed a model[3] to identify the bulk temperature associated to bubble departure (T_{dep}). The model consist in evaluating Péclet[1] number and selecting the appropriate correlation:

$$\begin{cases} Pe < 70000 \rightarrow T_{dep} = T_{sat} - 0.0022 \frac{q''_s D_h}{k_l} \\ Pe > 70000 \rightarrow T_{dep} = T_{sat} - 153.4 \frac{q''_s}{c_{p,l} k_l} \end{cases} \tag{2}$$

The cross-section average void fraction is computed according to the Drift flux model:

$$\bar{\alpha} = \frac{\bar{x}_v}{C_0 + \frac{\bar{u}_{gj}}{J}} \tag{3}$$

Drift velocity is obtained by the following relation[3]:

$$\bar{u}_{gj} = 2.9 \left[\sigma g \left(\frac{\rho_l - \rho_g)}{\rho_l^2} \right) \right]^{0.25} \tag{4}$$

Where the liquid and vapor densities are evaluated at saturation conditions. Finally the void fraction is obtained by considering the maximum between two relation: The first one adopt

[1] In this work is evaluated the average Péclet number between the onset of nucleate boiling and the saturation point

$C_0 = 1$ [2] and use a modified vapor quality:

$$x'(z) = x_{th}(z) - x_{th}(z_{dep}) exp\left[\frac{x_{th}(z)}{x_{th}(z_{dep})} - 1\right] \tag{5}$$

wehre $x_{th} = (h_b(z) - h_l)\,/h_{lg}$ is the thermodynamic equilibrium quality. This correlation is able to describe the void fraction before the saturation point but at the end of the two phase region give wrong results, for this reason is introduced the second model, which adopt x_{th} instead of x' and use a function for C_0 which depends on the volume quality $\bar{x}_v$:

$$C_0 = \bar{x}_v \left[1 + \left(\frac{1}{\bar{x}_v} - 1\right)^{(\rho_g/\rho_l)^{0.1}}\right] \tag{6}$$

In the figure below is reported a result for a fluid with mass flow rate equal to 0.2 kg/s in pipe with a diameter $D = 0.0119\ m$.
A vertical line identify the point at which the bulk is at saturation.

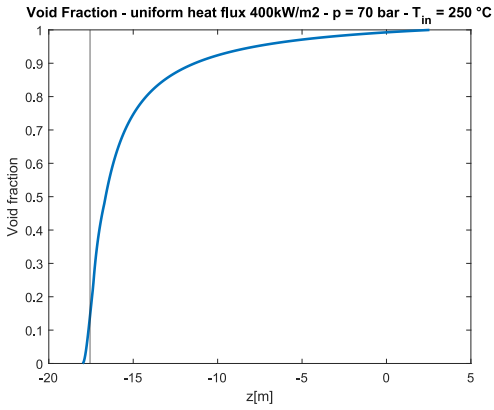

Figure 2: *Cross-section average void fraction axial profile*

III. Correlations for heat transfer coefficients

A flow boiling system can be devided in five regions having different formulation for the heat transfer coefficient.

[2] In principle value lower than one should be adopted since void fraction is higher at the wall then at the core in the subcooled boiling region

i. Subcooled liquid region

For subcooled water the Dittus-Boelter correlation is adopted, α_l can be computed through the Nusselt number[3]:

$$\begin{cases} Nu_{DB}(T) = 0.023 Re(T)^{0.8} Pr(T)^{0.4} \\ \alpha_l(T) = k_l(T) Nu_{DB}(T)/D_h \end{cases} \tag{7}$$

Since fluid properties depends on temperature the heat transfer coefficient change with the heating of the fluid.

ii. Subcooled boiling region

For the subcooled boiling region a Rohsenow type approach is adopted:

$$(q''_s)^2 = (q''_{s,FC})^2 + (q''_{s,NB} - q''_{s,s})^2 \tag{8}$$

where $q''_{s,FC} = \alpha_{lo}(T_{s,z} - T_{b,z})$ is the heat flux considering the "liquid only" forced convection, α_{lo} is computed with the Dittus-Bolter correlation.
$q''_{s,s}$ is the heat flux at the onset of nucleate boiling (ONB) and $q''_{s,NB}$ is the heat flux in the nucleate boiling region (NB).
The criterion adopted to select the correct correlation for this region consist in observing the temperature discontinuity at the transition with the saturated boiling region, the smaller is the temperature difference the better is the correlation. In particular two correlation have been analyzed: the Rohsenow NB correlation and the Mostinski model with Palen correction. According to the criterion stated before, Mostinski correlation has been chosen:

$$q''_{s,NB} = (0.1011 p_c^{0.69} 1.8 p_r^{0.17})^{1/0.3} (T_s - T_{sat})^{1/0.3} \tag{9}$$

Where p_c and p_r are respectively the critical and the reduced pressure.

iii. Saturated boiling region

This region start at liquid saturation and end at the critical heat flux (CHF). The Kandlikar[3] model has been adopted [5]:

$$\frac{\alpha_{tp}}{\alpha_l} = C_1 C_o^{C_2} + C_3 B_o^{C_4} F_{fl} \tag{10}$$

[3] Formulation for vertical tubes

Where $C_0 = ((1-\bar{x}_{th})/\bar{x}_{th})^{0.8}(\rho_g/\rho_l)^{0.5}$ is the convection number and $B_o = q''_s/(Gh_{lg})$ is the boiling number.
α_l is the "liquid alone" heat transfer coefficient[4]. The correlation provide two set of constants one for the boiling region and one for the convective one, the heat transfer is the higher obtained with the two set of constants. Finally $F_{fl} = 1$ is a parameter which depend from the fluid.

iv. Liquid deficient region

From the Dryout to the complete water evaporation, Groeneveld correlation is applied[3][5]. This model relate the "vapor only" Nusselt number with the mass quality and the fluid properties at the wall temperature, as reported in equation 11.

$$Nu_{go}(z,T_s) = a\left\{Re_{go}\left[\bar{x}_{th}(z) + \frac{\rho_g}{\rho_l}(1-\bar{x}_{th}(z))\right]\right\}^b Pr_{g,s}(T_s)^c Y^d \quad (11a)$$

$$Y(z) = 1 - 0.1\left(\frac{\rho_g}{\rho_l}-1\right)^{0.4}(1-\bar{x}_{th}(z))^{0.4} \quad (11b)$$

Properties are evaluated at saturation condition, except for $Pr_{g,s}(T_s)$ which depend from wall temperature. Since the wall temperature is an unknown an iterative procedure is needed: The code start with a guess value for temperature, then it compute water properties near the heating surface, and so the heat transfer coefficient, from which a better estimation for $T_s(z)$ is achived.

v. Superheated vapor region

This region is characterized by single phase vapor, then Dittus-Bolter correlation has been adopted.

[4]liquid flowing alone with it's apparent mass flux

[5]Two set of constants are considered, one for tubes, one for annular pipes. The latter is used for rod bundles

IV. ONB and CHF positions

i. Onset of nucleate boiling

The code require as input the critical cavity radius r_c, the procedure to individuate the ONB is based on Hsu's criterion, which consist in equating liquid and the bubble temperature at the distance r_c from the wall:

$$\begin{cases} T_{l,c} = T_{s,c} - \frac{q''}{k_l} r_c \\ T_{g,c} = T_{sat} + \frac{2\sigma T_{sat}}{\rho_g h_{lg} r_c} \end{cases} \quad (12)$$

The required wall superheating at ONB results[6][7]:

$$T_{s,ONB} - T_{sat} = \left(\frac{q''_{s,ONB} r_c}{k_{l,sat}} + \frac{2\sigma T_{sat}}{\rho_g r_c h_{lg}}\right) Pr \quad (13)$$

Then the nucleate boiling starts when the surface temperature reach $T_{s,ONB}$.

ii. Dryout critical heat flux

The starting point of the liquid deficient region is determined by the Bowring's model with a correction which take into account the non uniformity of the heat flux [1]:

$$q''_{s,CHF} = \frac{A + B(h_{sat} - h_{in})}{C + z_{CHF} Y_c} \quad (14)$$

A,B and C are functions of the pressure [3][2], and Y_c is the correction factor defined as

$$Y_c = \frac{1}{z_{CHF}} \int_0^{z_{CHF}} \frac{q''_s(z)}{q''_s(z_{CHF})} dz \quad (15)$$

To solve such an equation an iterative procedure is required, starting from a guess value for z_{CHF} is possible to compute the correction factor and so the heat flux, at which is associated a better estimation for z_{CHF}.

V. Correction of the discontinuities

As you can see in figure 3 at the beginning and at the end of the two phase region the

[6]Correction by Frost - Dzakovic with Prandtl number

matching of the heat transfer coefficient is far from being perfect, this lead to non physical discontinuities in the wall temperature.

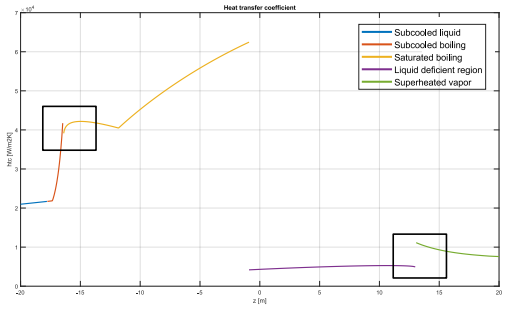

Figure 3: *Heat transfer coefficient along the pipe*

To manage this problem a linear extrapolation of the temperature from the anterior region is performed until the exact matching is achived, as reported in figure 4.

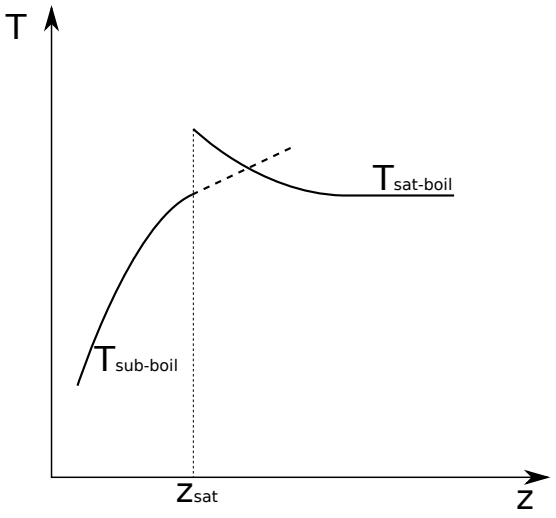

Figure 4: *extrapolation procedure*

In particular, for the transition in superheated vapor, a slightly different procedure is needed, the mixture temperature is linearly extrapolated, then a weighted average between it and vapor temperature is performed. The weighting function $f(z)$ depend on the position in such a way that the vapor contribute increase with the z coordinate[7]:

[7]this procedure is needed to reduce the temperature overestimation

$$T_b = T_{extrap} f(z) + T_{b,vapor}(1 - f(z)) \qquad (16)$$

VI. Results

In this section are presented some results obtained with different power distributions with same maximum heat flux and same flow properties. Power profiles are reported in table 1:

Figure 5	$q''_s = 500\ kW/m^2$
Figure 6	$q''_s = \frac{500}{H_a}(z + H_a/2)\ kW/m^2$
Figure 7	$q''_s = 500cos(\pi \frac{z}{H_a})\ kW/m^2$

Table 1: *Heat flux axial distribution*

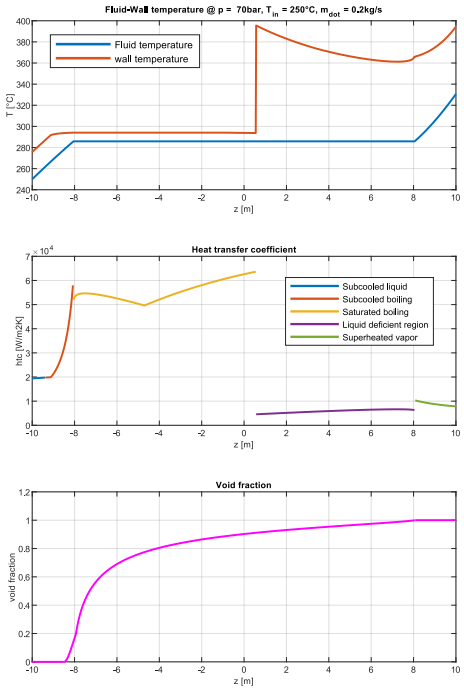

Figure 5: *Result from uniform heat flux axial profile*

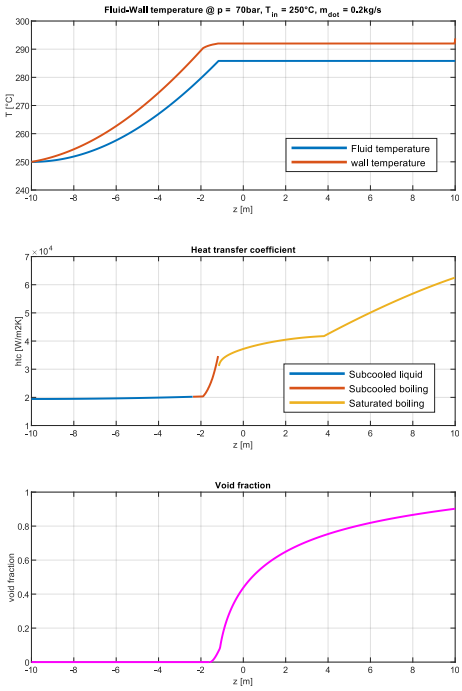

Figure 6: *Results from linear heat flux*

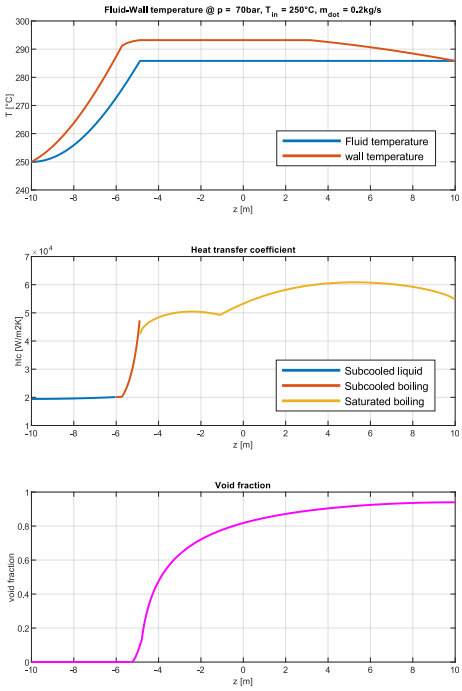

Figure 7: *Results from cosine shaped heat flux*

VII. How to use

The user should first decide if he want to simulate a pipe or a single channel of a rod bundle. According to this choice, file *data_pipe.m* or *data_rod.m* must be modified with the desired geometry, heat flux and inlet conditions. In these files the variable *"uniform"* state if q''_s is uniform (*uniform = 1*) or non uniform (*uniform = 0*). In *MAIN.m* the variable *"rod"* must be modified as well: if the geometry is a rod bundle the user have to put *rod = 1*, if not *rod = 0*.

VIII. Comments

For what concern nuclear plants, with some modification this code can be coupled with neutronics and fuel thermal analysis, for example adopting the scheme in figure 8:

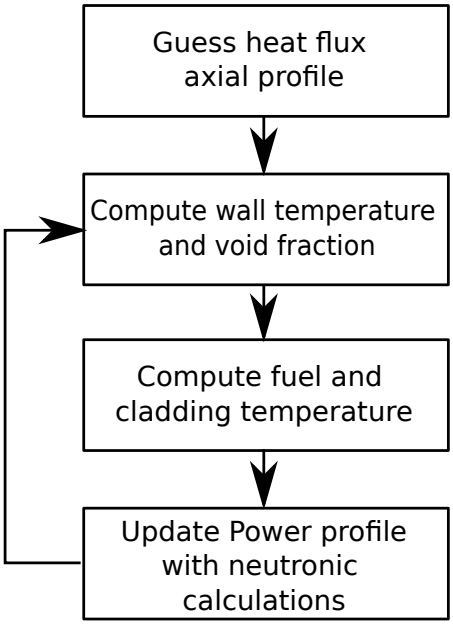

Figure 8: *hypothetical scheme for fuel performance code*

It could be interesting in the analysis of

a nuclear reactor core to consider pressure drops in order to obtain a more accurate fluid description, So further development can be done in this direction. In appendix A i propose a simple approach to accomplish this task.

One additional point could be implementing the possibility to analyze channel with non uniform diameter, since heat transfer coefficient depend also by mass flux.

In the end, the validity of the results obtained by this code depends from the range of applicability of the correlations involved, but also from intrinsic limitations. For example too high heat fluxes bring the coolant to temperatures much higher than the capability of the function XSteam.
Moreover it's observed that the temperature matching at the end of the two phase region is not achieved if the vapor is just slightly superheated, and also with particular shape of the heat flux, a better interpolation procedure is needed.

A. Appendix: Pressure drop correction

In order to evaluate pressure drops one should couple momentum and energy equations. For simplicity, i considered the homogeneous flow model, this simplify a lot the description since permit to us to rely just on bulk properties. Considering a constant passage area, momentum and energy equations for the two phase region result to be:

$$\begin{cases} \frac{dp}{dz} = \frac{dp_f}{dz} - G^2 \frac{d\bar{v}_b}{dz} - g\bar{\rho}_b sin\theta \\ \frac{dh_b}{dz} = \frac{1}{\bar{\rho}_b}\left[\frac{dp}{dz} - \frac{dp_f}{dz}\right] + q''_s \frac{S}{\Omega G} \end{cases} \tag{17}$$

A simple algorithm to solve such a system could be the one in figure 9 which start from the solution of equation 1.
Note that for single phase regions is possible to use the same system of equations just considering the single phase density, viscosity and friction.

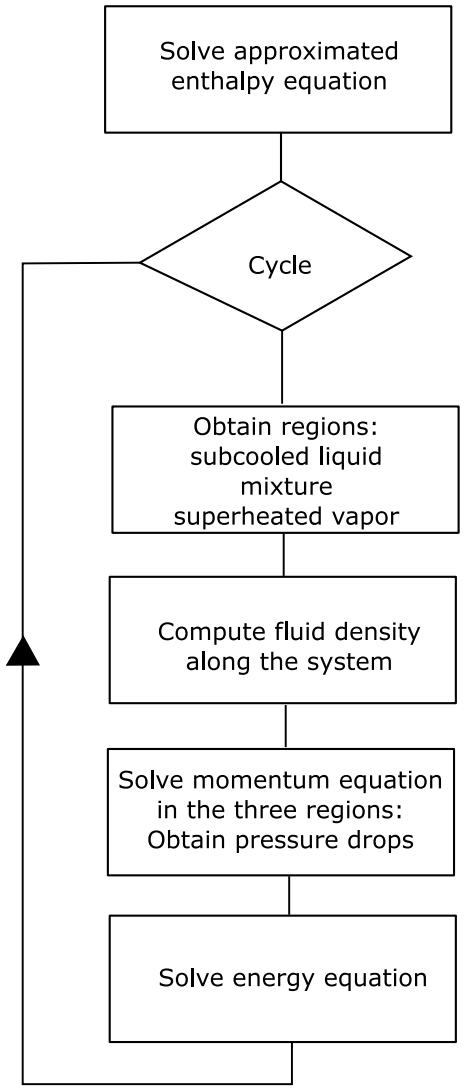

Figure 9: *hypothetical scheme for the solution of system 17*

References

[1] M. Ferrouk, S. Aissani, F. D'Auria, A. Del-Nevo, A. Bousbia Salah. Assessment of 12 CHF prediction methods, for an axially non-uniform heat flux distribution, with the RELAP5 computer code. Volume 238, Issue 10, 2008, Pages 2718-2725.

[2] Caleb Jernigan. Critical heat flux model improvement in ctf for natural circulation type reactor, Thesis 2016, The Pennsylvania State University.

[3] Neil E. Todreas, Mujid S. Kazimi. Nuclear systems volume I, thermal hydraulic fundamental. CRC press, second edition.

[4] Mirza M. Shah, A new correlation for heat transfer during boiling flow through pipe, ASHRAE Transactions, 1976.

[5] Kandlikar, Satish. (1990). A General Correlation for Saturated Two-Phase Flow Boiling Heat Transfer Inside Horizontal and Vertical Tubes. Journal of Heat Transfer-transactions of The Asme. J HEAT TRANSFER. 112. 219-228. 10.1115/1.2910348.

[6] R. T. Lahey, Jr.,F. J. Moody. The Thermal-Hydraulics of a Boiling Water Nuclear Reactor. American Nuclear Society La Grange Park, Illinois USA, second endition.

[7] Clayton T. Crowe. Multiphase Flow Handbook. CRC Taylor and Francis.